Math Game III: Electoral College Mechanics

Comparative Electoral Politics

Session 3, Vanderbilt Summer Academy 2017
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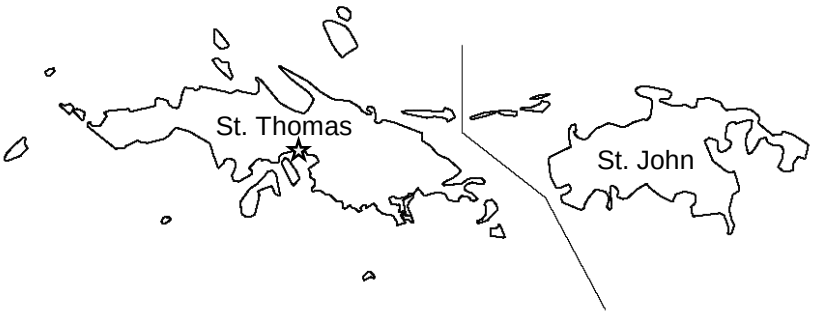
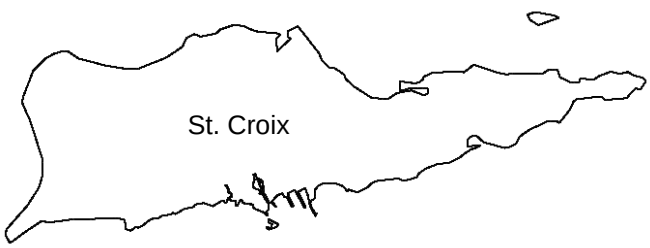
As noted during lecture, the U.S. Electoral College is a method of *indirect* election: the people vote *directly* for *electors* to the College, which in turn (and once convened), votes *directly* for presidential candidates. Each state is apportioned a number of electors to the College equivalent to the size of its congressional delegation. (Because the U.S. House is based upon population, the Census' decennial reapportionment of House seats—due to population changes—also affects a state's number of electors.)

The mechanical operation of the Electoral College is confusing because it is convoluted. In this math game, we will explore how it works, by sailing south to the Caribbean.

U.S Virgin Islands





The Set-up:

Let's pretend that the Virgin Islands (V.I.), a Caribbean territory the U.S. purchased from Denmark in 1917, is newly sovereign and is scheduled to elect its first ever president. Additionally, and due to its historical association with the U.S., it has decided to adopt its own electoral college, comprised of 26 electors, with each of the three main islands—St. Croix, St. John, and St. Thomas—serving as a state. (Suppose the size of the V.I. House is 20 seats, whereas the size of the V.I. Senate is 6 seats; the House is based upon the *population principle*, while the Senate utilizes *equal representation*.)

Instructions to Part 1: Apportionment of V.I. Electors to the States

Below, you will find the formula that the U.S. uses to apportion U.S. House seats (and, thus, electors to the College, as well) among the states: the Method of Equal Proportions (MEP). Let's use MEP to apportion our 20 fictitious V.I. House seats among the 3 states, so that we can arrive at the size of each state's V.I. Electoral College representation.

Method of Equal Proportions (MEP)
$$A_{nj} = \frac{P_j}{\sqrt{n_j(n_j+1)}}$$

As a first step, MEP awards to each state 1 seat. The remaining 17 seats are allocated, one at a time, to states based on *priority number, A*. (The fourth seat, for example, goes to the state with the highest *A* overall, the fifth seat to the second highest *A* overall, etc.)

We can use the formula above to generate the table of priority numbers we need to carry out the apportionment process. In the formula, P_j = state *j*'s Population, while n_j = the Number of Seats state *j* holds before the *next* seat is allocated.

Part 1, Section 1: The Apportionment Process

1) Complete the table using MEP. In rows 3-12, write in the priority number (A_{nj}) for each box. Next, <u>circle</u> the 17 highest priority numbers; write the totals in row 13.

State	St. Croix	St. John	St. Thomas
Population	50,600	4,200	51,600
1st priority number (use n = 1)		2,969.8	
2nd priority number (use n = 2)			21,065.6
3rd priority number (use n = 3)	14,607.0		
4th priority number (use n = 4)		939.1	11,538.1
5th priority number (use n = 5)		766.8	
6th priority number (use n = 6)			
7th priority number (use n = 7)	6,761.7		
8th priority number (use n = 8)			
9th priority number (use n = 9)	5,333.7		
10th priority number (use n = 10)		400.5	4,919.9
Total Seats (# of circles + 1)			

2) Calculate each state's total V.I. Electoral College representation:

a) St. Croix: _____ House seats + 2 Senate seats = _____ V.I. electors.

b) St. John: _____ House seats + 2 Senate seats = _____ V.I. electors.

c) St. Thomas: _____ House seats + 2 Senate seats = _____ V.I. electors.

Part 1, Section 2: State Over- or Underrepresentation

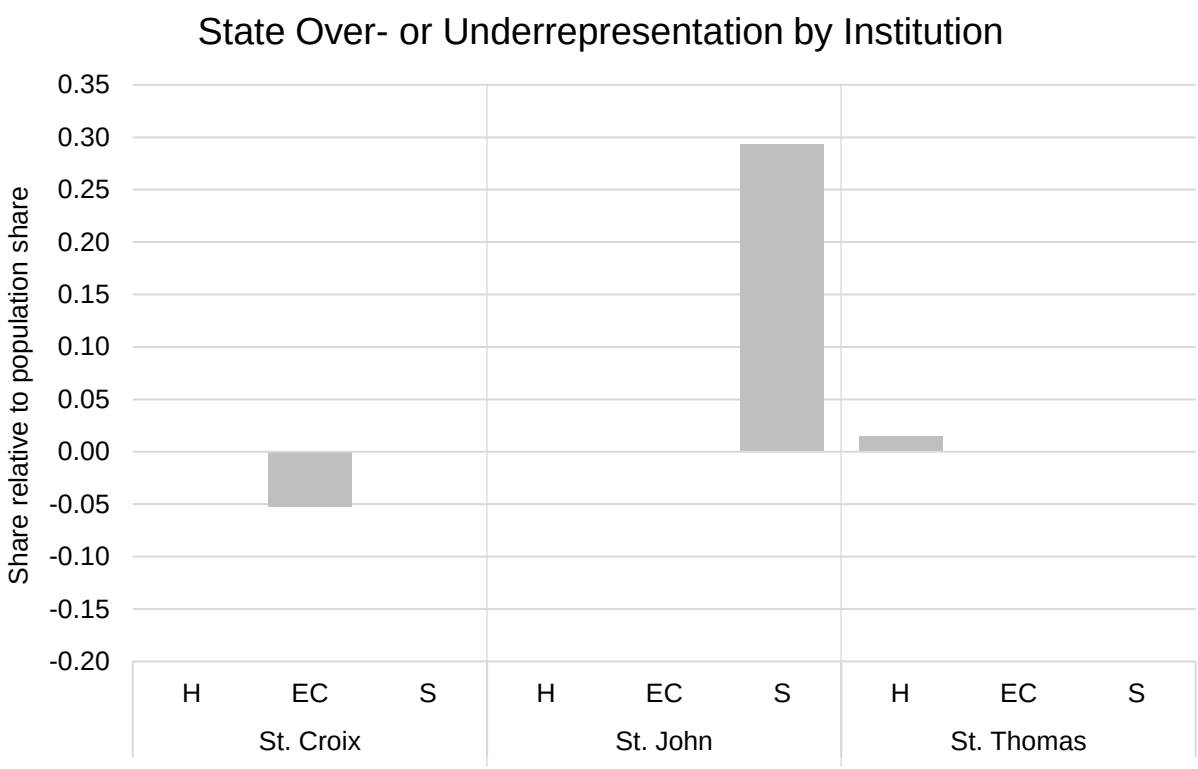
It's important for us to assess the extent to which an institution—V.I. House, V.I. Senate, and V.I. Electoral College—over- or underrepresents a state relative to its population. States that are overrepresented have *less* power in the institution than their population share would predict, whereas those that are underrepresented have *more* power.

Over- and Underrepresentation
$$r_{ji} = \frac{S_j}{\sum_{j=1}^{n} S_j} - \frac{P_j}{\sum_{j=1}^{n} P_j}$$

We can use the formula (left) to obtain an institution's over- or underrepresentation, *r*, of each state *j*, where, P_j = state *j*'s Population and S_j = state *j*'s Number of Seats in institution *i* {*i* = House, Electoral College, and Senate}.

3) First, complete the table using the formula above. Second, plot each value r_{ji} in the bar graph below. (Note: H = House, EC = Electoral College, S = Senate.)

State	St. Croix	St. John	St. Thomas
Population (r_{jP})	0	0	0
V.I. House (r_{jH})			0.015
V.I. Electoral College (r_{jEC})	- 0.052		
V.I. Senate (r_{jS})		0.294	



4) **Do positive values of r_{ji} indicate over- or underrepresentation of state *j* in institution *i*? What about negative values? Why do the cells in the population row each contain a 0?**

5) **Which island state is most overrepresented across the institutions? Why? In which institution is this overrepresentation the greatest? Why? What would this state's representation be if MEP did not guarantee every state at least one seat?**

6) **What do you observe about the r_{ji} values for the V.I. Electoral College relative to those for the V.I. House and the V.I. Senate? What does this tell us?**

Instructions to Part 2: Allocation of V.I. Electors to the Candidates

Now that we've apportioned the electors among the states, it's time to run the presidential election! Suppose that each state uses *winner-take-all unit rule* to allocate its electors to the candidates. Let's further pretend that two major parties, the centre-left Labour (L) and centre-right Unionist (U) parties, dominate the nascent V.I. party system. However, in St. Croix, a leftist, separatist minor party, Free Croix (FC), has made the ballot; it advocates for an independent St. Croix and has proved to be a thorn in the side of Labour.

In this section, we will not only run the actual election (via the V.I. Electoral College), but also examine what might have happened had we used 3 alternatives to the status quo.

Part 2, Section 1: The Electoral College with Unit Rule Allocation

1) Complete the table and graph below using the election data provided.

State		St. Croix	St. John	St. Thomas	Total
V.I. College Electors			3		26
Population		50,600	4,200	51,600	106,400
Voting Age Population		26,500	3,300	39,500	69,300
Registered Voters		23,000	2,800	34,000	59,800
Total Voters		18,000	2,000	25,000	45,000
Turnout (%)		78.3%			
Votes disaggregated by party					
Labour (L)	Votes	6,660	980	14,500	22,140
	%				
Unionist (U)	Votes	8,640	1,020	10,500	20,160
	%	48%			
Free Croix (FC)	Votes	2,700	0	0	2,700
	%	15%		0%	6%
Official Declaration (*of state winners*)					
State Electoral Vote Winner			Unionist		

2) Aggregate election result: Fill in the following blanks.

- Labour (L): a) Total share (%) of the popular vote is _____ %.
  b) Total electors is _____ (share of V.I. College is 46.2%).

- Unionist (U): a) Total share (%) of the popular vote is _____%.
  b) Total electors is _____ (share of V.I. College is _____ %).

- Free Croix (FC): a) Total share (%) of the popular vote is 6%.
  b) Total electors is _____ (share of V.I. College is _____ %).

- V.I. College decision rule = simple majority (50% + 1 elector) = _____ electors.

- The _____ Party has passed the decision rule and won the election.

3) Has the party that won the electoral vote failed to win a plurality of the aggregate popular vote (i.e., is there a so-called 'wrong winner' president)?

Part 2, Section 2: The Electoral College with Proportional Allocation of Electors

Using the same electoral data provided in Part 1, Section 1, let's allocate each state's electors to candidates according to the first reform alternative, the Proportional Plan. Under this plan, a state allocates its electors not as a winner-take-all unit to a single party, but rather among the parties in *proportion* to their share of the *state popular vote*. As before, the candidate with a majority of College electors, wins the presidency.

Hare
$Q_j = \frac{V_j}{M_j}$

Because electors (as humans!) can't (legally) be cleaved into pieces, fractional remainders are not permitted. To solve this problem, recall the PR allocation formulas we learned in Math Game I. Let's use the simplest method: the Hare Quota. In the formula (left), Q_j = the Quota for state j; V_j = Total Votes Cast in state j; and, M_j = Number of Electors Available in state j.

4) Let's start with St. Croix. Based on the election data, what is its Hare Quota?

5) Complete the table for St. Croix using its Hare Quota.

Party	Labour (L)	Unionist (U)	Free Croix (FC)
Votes (V_j)	6,660	8,640	2,700
Votes (V_j) ÷ Quota (Q)	4.03		
a) Initial Seats Awarded			1
Votes Used (“Spent”)		8,180	
Votes Remaining		460	
Remainder Rank		2nd	
b) Final Seats Awarded			
Total Seats (a + b)			

6) Let’s move to St. John. Based on the election data, what is its Hare Quota?

7) Complete the table for St. John using its Hare Quota.

Party	Labour (L)	Unionist (U)	Free Croix (FC)
Votes (V_j)	980	1,020	0
Votes (V_j) ÷ Quota (Q)	1.47		0
a) Initial Seats Awarded		1	0
Votes Used (“Spent”)			0
Votes Remaining	314		0
Remainder Rank			3rd
b) Final Seats Awarded			0
Total Seats (a + b)			0

8) Let’s finish with St. Thomas. Based on the election data, what is its Hare Quota?

9) Complete the table for St. Thomas using its Hare Quota.

Party	Labour (L)	Unionist (U)	Free Croix (FC)
Votes (V_j)	14,500	10,500	0
Votes (V_j) ÷ Quota (Q)	6.96		0
a) Initial Seats Awarded	6		0
Votes Used ("Spent")			0
Votes Remaining		85	0
Remainder Rank			3rd
b) Final Seats Awarded			0
Total Seats (a + b)			0

10) Aggregate election result: Fill in the following blanks.

- Labour (L): Total electors is _____ (share of V.I. College is _____ %).
- Unionist (U): Total electors is _____ (share of V.I. College is _____ %).
- Free Croix (FC): Total electors is _____ (share of V.I. College is _____ %).
- V.I. College decision rule = simple majority (50% + 1 elector) = _____ electors.
- The _____ Party has passed the decision rule and won the election.

11) Has the party that won the electoral vote failed to win a plurality of the aggregate popular vote (i.e., is there a so-called 'wrong winner' president)?

Part 2, Section 3: The Electoral College with District Allocation

Below, you will find the same election data as we used in Sections 1–2, but disaggregated by V.I. House district; this is the level of granularity we need for the second reform alternative, the District Plan. Under this plan, a state allocates its electors as follows: first, the *district popular vote plurality winner* earns the 1 elector tied to that V.I. House district; and, second, the remaining 2 electors are awarded 'at-large' as a unit to the candidate who wins the *aggregate state* popular vote. As before, the candidate with a majority of College electors, wins the presidency.

12) In each district, circle the most votes; write the name of the party in column 5.

State & District No.		Labour (L)	Unionist (U)	Free Croix (FC)	District Winner	At-Large Winner
St. Croix	D1	1,100	1,000	300		
	D2	750	1,300	250	Unionist	
	D3	650	900	350		
	D4	460	690	300		
	D5	850	750	200		
	D6	800	900	200		
	D7	1,050	950	200	Labour	
	D8	400	1,400	300		
	D9	600	750	600		
St. John	D1	980	1,020	0		
St. Thomas	D1	1,700	850	0		
	D2	1,250	1,300	0		
	D3	1,350	1,400	0		
	D4	1,050	1,100	0		
	D5	1,600	800	0		
	D6	1,700	950	0		
	D7	1,350	1,400	0		
	D8	1,900	850	0		
	D9	1,400	600	0	Labour	
	D10	1,200	1,250	0		

13) Aggregate election result: Fill in the following blanks.

- Labour (L): Total electors is _____ (share of V.I. College is _____ %).
- Unionist (U): Total electors is _____ (share of V.I. College is _____ %).
- Free Croix (FC): Total electors is _____ (share of V.I. College is _____ %).
- V.I. College decision rule = simple majority (50% + 1 elector) = _____ electors.
- The _____ Party has passed the decision rule and won the election.

14) Has the party that won the electoral vote failed to win a plurality of the aggregate popular vote (i.e., is there a so-called 'wrong winner' president)?

Part 2, Section 4: The Electoral College as reformed by NPVIC

Finally, let's conclude our calculations with a third alternative plan: the National Popular Vote Interstate Compact. As noted during lecture, states decide whether or not to join the compact; if they do, then they pledge to award all of their electors as a winner-take-all unit to the winner of the *aggregate national popular vote*, rather than their state popular vote winner (if different). The compact, a latent or conditional reform, 'activates' once member states collectively comprise an Electoral College majority. By design, NPVIC prevents wrong winner presidents, because the winner of the national popular vote is guaranteed a majority of electors.

Suppose that two states, St. John and St. Thomas, have joined NPVIC, whereas St. Croix as not done so. (The independence-seeking Free Croix Party opposes all compacts with the other states and has successfully stymied the NPVIC legislation in St. Croix.)

15) Aggregate election result: Fill in the following blanks.

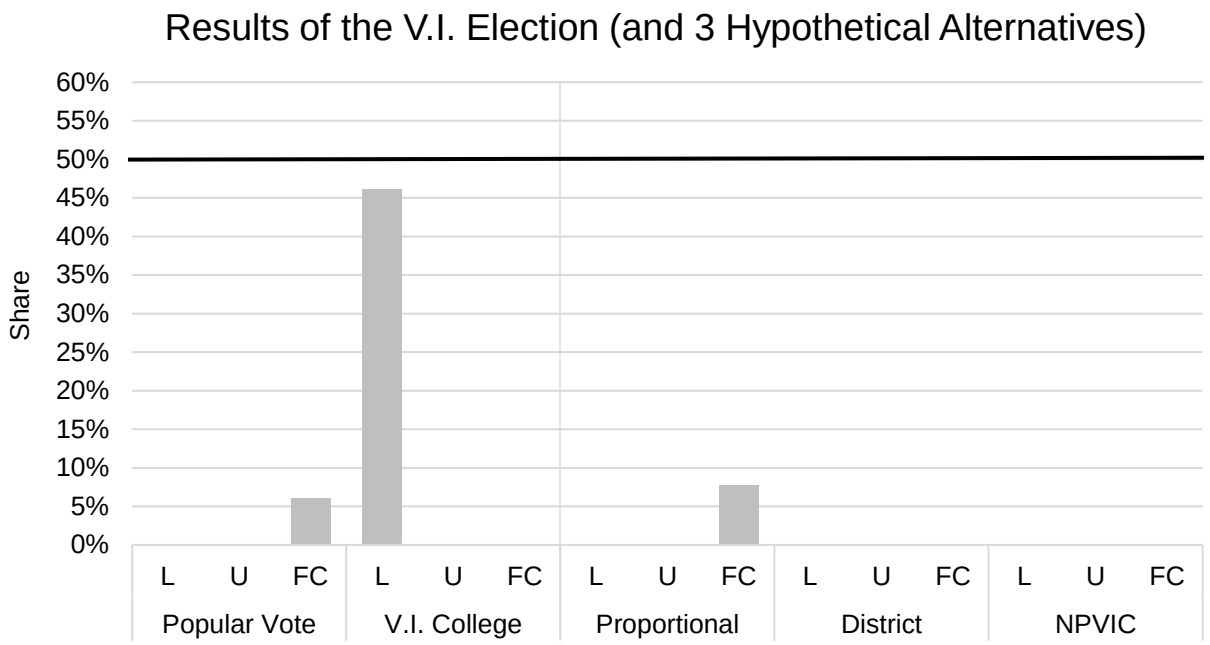
- The compact member states comprise _____ electors to the V.I. College.
- V.I. College decision rule = simple majority (50% + 1 elector) = _____ electors.
- The compact is thus _____ (Pick ‘active’ or ‘inactive’).
- Labour (L): Total electors is _____ (share of V.I. College is _____ %).
- Unionist (U): Total electors is _____ (share of V.I. College is _____ %).
- Free Croix (FC): Total electors is _____ (share of V.I. College is _____ %).
- The _____ Party has passed the decision rule and won the election.

16) Has the party that won the electoral vote failed to win a plurality of the aggregate popular vote (i.e., is there a so-called ‘wrong winner’ president)?

Instructions to Part 3:

Great job! Now that you have calculated the result of the presidential election, as well as the hypothetical results of the three reform alternatives (the Proportional Plan, the District Plan, and NPVIC), plot your results for each in the appropriate graph below. Additionally, you should also plot each party’s share of the aggregate popular vote (which you obtained in Part 2, Section 1, Question 2 on page 6). I have provided three of the 15 bars for you.

After filling in the missing bars for the graph, I would like you to answer a few conceptual questions about the math you completed and the results you obtained for Part 2.



1) **Do any of the four systems above scrap the so-called 'federalism principle' (i.e., that each state should be an equal member of the Senate)? If so, which ones? If not, what are the 'costs' of adhering to this principle?**

2) **Looking back at actual the election result on page 6, what are three possible factors that contributed to the wrong winner result? (Hint: One of the three can be found in the question you just answered).**

3) **You should have also found that one of the 3 hypothetical reform alternatives generated an inconclusive result (i.e., no candidate won a majority of the V.I. College). First, name the system. Second, what was the (main) reason that this result occurred? Should we thus view this reform alternative with suspicion?**

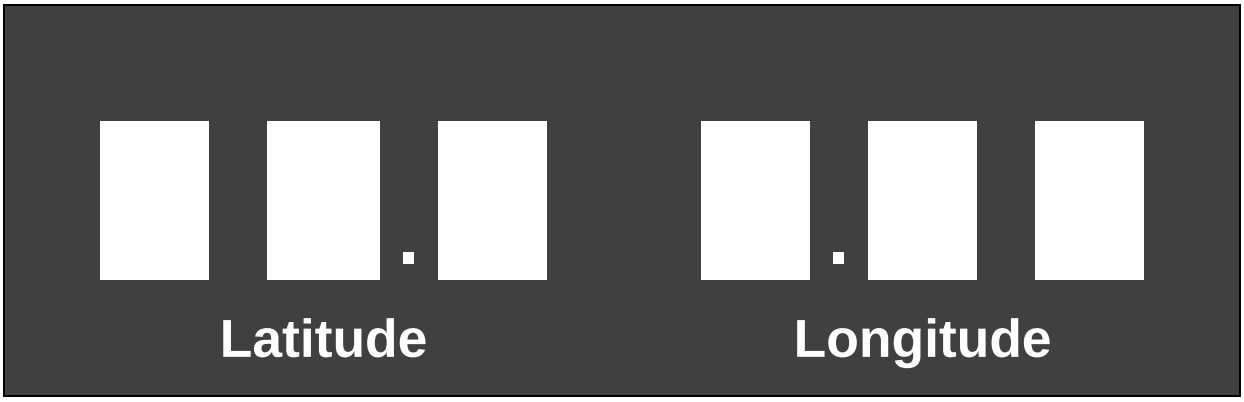
4) **Looking back at the District Plan, why was Labour's share of total electors so far below its share of the aggregate popular vote? (Hint: Look at St. Thomas and, to a lesser extent, St. Croix.) What does this tell us about the importance of district lines under this plan? Would the urge to gerrymander get worse?**

5) **One reform alternative that we didn't explore in this math game is the Direct Popular Vote Plan, in which the Electoral College is abolished. In which ways does this plan solve the problems four systems you identified above? What do you think the result of the election would have been under this system?**

Instructions to Part 3 (cont.):

Now it's time to unlock the code! To do so, add together the total number of electors you obtained for the Unionist Party across the four electoral systems for which we performed calculations; this will produce the first two numbers in the code. To produce the third number in the code, add together the three V.I. Senate over-/underrepresentation shares (r_{js}). (Be sure to pay attention for negative signs!) Finally, and to obtained the final three numbers, take the absolute value of the difference obtained by subtracting from the Hare Quota for St. Thomas *both* the Hare Quota for St. John and the Hare Quota for St. Croix.

Write the resulting 6-digit number in the blanks below. These numbers correspond to GPS coordinates located within a particular country. Using Google Maps (or just Google search), use the coordinates to locate the country (be sure to retain the decimal points).



The coordinates are located within the country _____.

When you think you have the correct country, find me and we'll see if you're right!